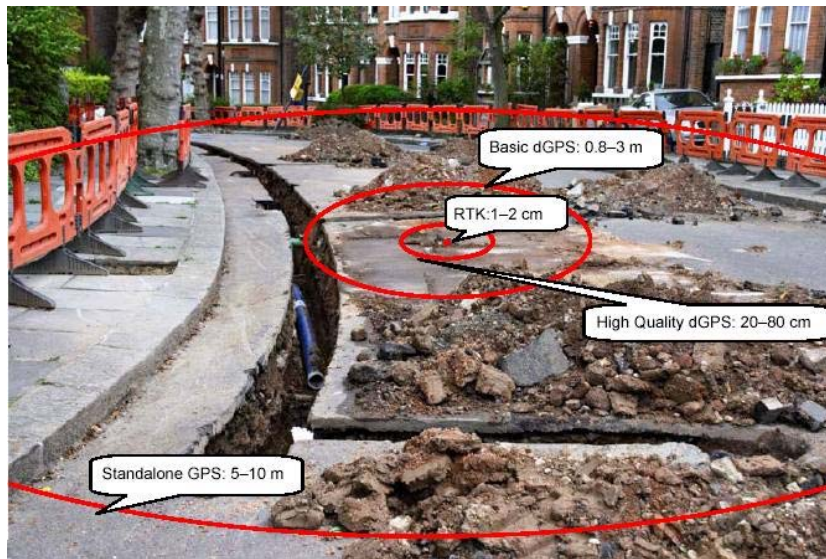




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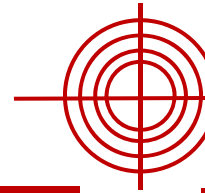
«A proposal of a statistical test to control positional accuracy by means of 2 tolerances simultaneously»

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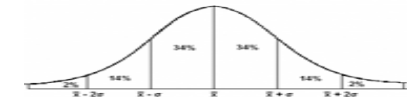
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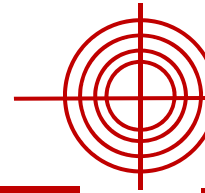


Introduction

- Spatial data are data referring to features that have a position in space, and for this reason positional quality is one of the most desirable characteristics of spatial data sets (SDS)
- Positional accuracy of SDS has traditionally been evaluated using control points. NMAS, EMAS, NSSDA, STANAG 2215, ASPRS, ...
- The majority of Positional Accuracy Assessment Methodologies (PAAMs) take as an underlying hypothesis the Gaussian distribution (Normal Distribution)
- It is also important to notice here that prior to any statistical analysis of the data assuming normality, the outliers must be eliminated.
- But several studies indicate that this hypothesis (normality of errors) is not true. In these cases, some other error's models have been proposed, as well as distribution-free models:

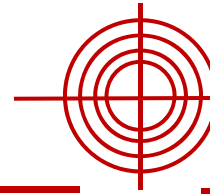


- LIDAR (Maune, 2007): Without a parametric model (distribution free)
- Manual digitizing (Bolstad et al 1990): Bimodal
- Digitizing (Tong & Liu, 2004): p-norm (Normal + Laplace)
- Geocodification (Cayo and Talbot 2003; Karimi and Durcik 2004, Whitsel et al. 2004): Log normal
- GNSS Observations (Wilson, 2006; Logsdon, 1995): Raleigh, Weibull
- Other models that are mentioned: Folded Normal, Half Normal, Gamma



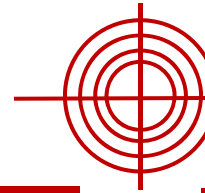
Introduction

- Nowadays we can access to very large data sets, and by very large computing capabilities, so we can use the observed distribution functions (Free-Distribution Functions or Parameters-Free-Distribution Functions) without major problems.
- The PAAMs based on Free-Distribution Functions are scarce in positional accuracy assessment. For instance, the use of percentiles is proposed for dealing with height data of DEM when capture (e.g. by Lidar) is performed on a vegetal cover.
- Some methods that can be applied to data belonging to free-distribution functions have been developed. A step further would require a statistical method allowing control in the same way (mean value, variation, control of outliers) as performed when data come from a Normal Distribution Function.



Introduction

- Our aim is to propose a general positional accuracy control method for dealing with data following any kind of distribution function.
- It is a method based on the observed distribution function of the data and can be applied to 1D, 2D or 3D data without the limitations of homogeneity of variances stated in traditional methods (e.g. EMAS, NSSDA, STANAG 2215, etc.).
- To achieve this aim we propose a method based on proportions of a multinomial distribution function in order to establish a strict control over data coming from any distribution function.
- The control is multiple and can test jointly proportions corresponding to tolerances related with, for instance, median values, extreme values (e.g. 95%), or the amount of outliers existing in the data set.
- The proposed control is based on an exact test, in the same way as the Fisher's exact test (Fisher 1922, Freeman and Halton 1951, Müller 2001).



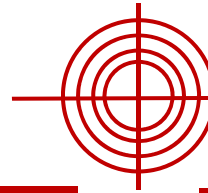
Approaching the idea

- In order to analyse a pass/fail model applied to positional accuracy control, an approximation is given in terms of the Binomial distribution (Ariza-López & Rodríguez-Avi, 2014).
- For each sampling point taken in a k-dimensional space, the value:

$$E_i = \sqrt{\sum_{i=1}^k (x_i - x_i^T)^2}$$

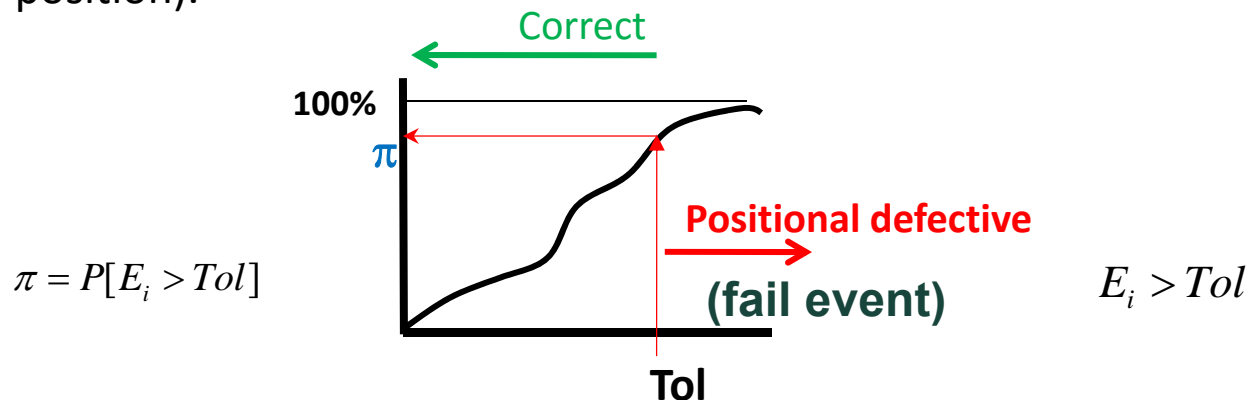
is calculated (where (x_1^T, \dots, x_k^T) is the “exact value”).

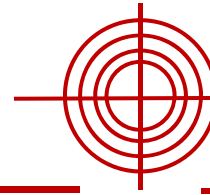
- Given a tolerance T , we count the number of sampling points where E_i is greater than T .
- This can be seen as a realization of a Binomial distribution with parameters N and p .



Approaching the idea

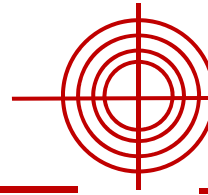
- The test is:
 - \mathbb{H}_0 : the proportion of defectives points is less or equal than p_0 , against
 - \mathbb{H}_1 : the proportion of defectives is greater than p_0 .
- \mathbb{H}_0 is rejected if $p = P[X > x | X \sim B(N, p_0)] < \alpha$ where x is the number of defectives (i.e. number of points whose error is greater than the previously specified tolerance) found in the sample.
- This approximation is a pass/fail procedure and implies that we are not able to distinguish the “degree of bad quality” of a defective point (in position).



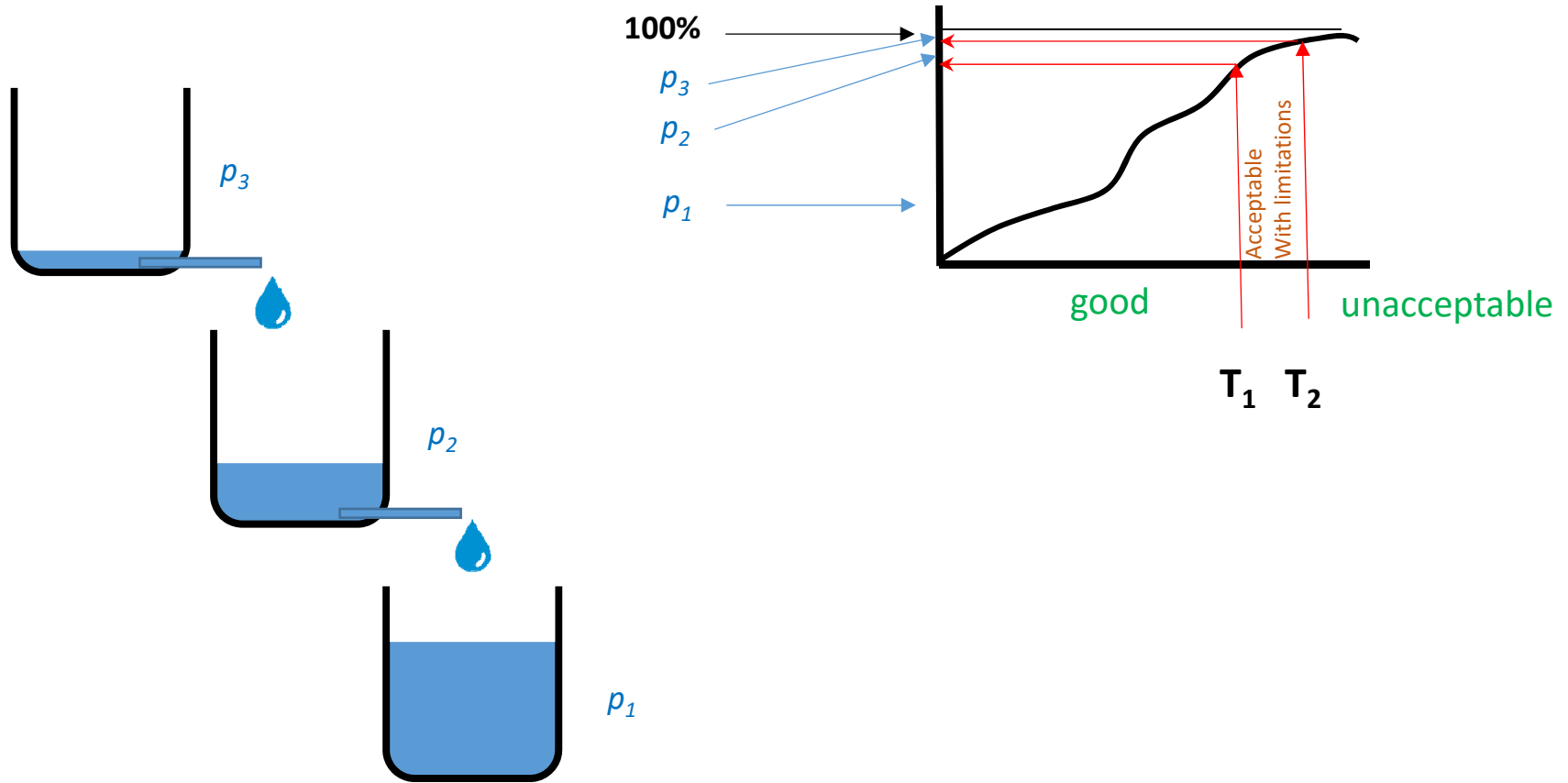


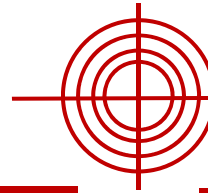
Proposed Method

- We propose a gradation in the degree of positional defectiveness, splitting the interval, the excess of error greater than the tolerance, into two parts through two metric tolerances
- Each interval defines a quality level, and we can determine the minimum percentage of “correct” points in the first level (best level), and the maximum percentage of points in each of the other two levels (worse levels).
- For instance, we can establish a percentage of at least a $p_1\%$ of points with an error less than a tolerance T_1 and at most a $p_2\%$ of points with an error between tolerances T_1 and T_2 ($T_2 > T_1$) and, in consequence, at most a percentage of $p_3\% = 100 - p_1\% - p_2\%$ of points with error greater than tolerance T_3 .
- So, we classify the sample into three (instead of two, as before) categories, and consequently, the multinomial approximation instead of the binomial approximation is adequate.



Proposed Method



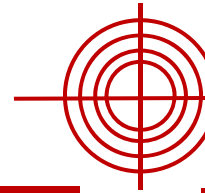


Proposed Method

- Multinomial distribution is a multivariate extension of a binomial distribution when we classify the result of an experiment into more than two categories.
- If we realize N independent experiments where we classify the results for exactly one of 3 categories, with probabilities π_1, π_2, π_3 , and $\pi_1 + \pi_2 + \pi_3 = 1$, then the mass probability function of a such multinomial $\mathcal{M}(N, \pi_1, \pi_2, \pi_3)$ is given by:

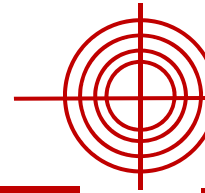
$$P[(X_1 = N_1, X_2 = N_2, X_3 = N_3)] = \binom{N!}{N_1! N_2! N_3!} \pi_1^{N_1} \pi_2^{N_2} \pi_3^{N_3}$$

- Where N_i is the number of points that belongs to the category i , which has a probability π_i .
- We use the multinomial distribution to propose an exact test to decide whether a specified hypothesis is false or not. Firstly we decide two tolerance levels, T_1 and $T_2, T_2 > T_1$ to decide if a point is
 - *adequate* ($E_i < T_1$),
 - *slightly inadequate* ($T_1 < E_i < T_2$) or
 - *roughly inadequate* ($E_i > T_2$).



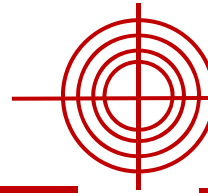
Proposed Method

- How to make the test:
 1. A sampling of size N is dropped, and we count the number of point that belongs in each category.
 2. The sampling statistics is $S = (N_1, N_2, N_3)$, so that $N_1 + N_2 + N_3 = N$, where N_i is the number of points that, in a sample of size N belongs to category i , $i = 1, 2, 3$
 3. Under the null hypothesis we can calculate the exact probability of obtaining a such point S and the probability of every point worse than S .
 4. As a result, to calculate the p-value we sum the probability, under the null hypothesis, of elements (M_1, M_2, M_3) that verify:
$$M_1 < N_1 \text{ or}$$
$$M_1 = N_1 \text{ and } M_2 < N_2$$
 5. We reject the null hypothesis if the p-value obtained is lesser than α .



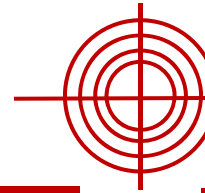
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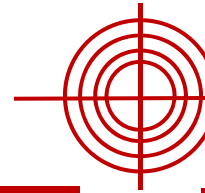
Proposed Method

- We can state for the two tolerances case that:
 1. The sampling statistics are: (N_1, N_2, N_3) , so that $N_1 + N_2 + N_3 = N$.
 2. Parameters of the multinomial distributions are:
$$N, \pi_1, \pi_2, \pi_3 = 1 - \pi_1 - \pi_2.$$
 3. The null hypothesis is:
$$\mathbb{H}_0: \text{Counting has a multinomial distribution with parameters } N, \pi_1^0, \pi_2^0, \pi_3^0 \text{ where } \pi_k^0 = P_k/100 \text{ and } \pi_1^0 + \pi_2^0 + \pi_3^0 = 1.$$
 4. The alternative hypothesis is:
$$\mathbb{H}_1: \text{At least one of these conditions: } \pi_1 < \pi_1^0 \text{ or } \pi_2 > \pi_2^0, \text{ or } \pi_3 > \pi_3^0, \text{ is true.}$$



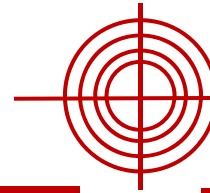
Simulation procedure

- This is an exact test, so the p-value is calculated as follows;
 1. Given (N_1, N_2, N_3) , we calculate the probability in the multinomial fixed by the null hypothesis to the obtained value and those counting of elements (M_1, M_2, M_3) that verify:
$$M_1 < N_1$$
$$M_1 = N_1 \text{ y } M_2 < N_2$$
 2. Adding up the p-value of all the cases that verify these conditions, rejecting the null hypothesis if the p-value obtained is lesser than α .
- To prove the validity of our proposal we develop two complementary simulations:
 - The first one is in order to prove the validity of the contrast with respect to the Type I error.
 - The second one is in order to prove the power of the contrast, that it is to say, the probability of rejecting the null hypothesis when this hypothesis is false.



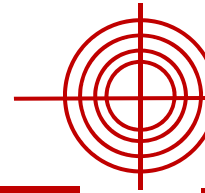
Simulation procedure

- First step
 1. We choose several values for N, the sampling size.
 2. For each value of N and a given value of $\pi^0 = (\pi_1^0, \pi_2^0, \pi_3^0)$, a space of events of a multinomial is generated, with all the corresponding probabilities. For each event the probability according to a multinomial $\mathcal{M}(N, \pi_1^0, \pi_2^0, \pi_3^0)$ is calculated.
 3. 2000 samples of three numbers coming from the above multinomial through a Montecarlo method are generated . For each one of the random samples the p -value is calculated according to the previously described procedure.
 4. The proportion of samples where the p -value is lesser than a fixed alpha is determined by counting.
 5. We repeat this procedure 50 times



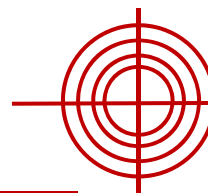
Simulation procedure

- Second step
 - To analyse the power of the contrast we proceed to a simulation
 - Sampling values are obtained from a multinomial distribution N, π^1 where π^1 is worse than the specification indicated by the null hypothesis. In consequence, the null hypothesis should be rejected.
 - So we calculate the probabilities of each simulated case under the null hypothesis, which is a multinomial N, π^0 .
 - In each case, simulations have been realized according to the simulation procedure described above.



Application example

- We show this procedure with three different examples.
 1. \mathbb{H}_0 is true, and we are going to see what happens if a sample confirms this hypothesis or not.
 2. The true model is worse, in the sense of generating a higher number of errors than the desired mode
 3. The sample refers to a situation which is better, in the sense of reduced errors, than that stated by \mathbb{H}_0 .
- Errors in X, Y and Z are distributed according to three $N(0, 1.5)$
- The quadratic error in each element is:
$$QE_i = x_i^2 + y_i^2 + z_i^2$$
which is Gamma(3/2, 4.5). In this case:
 - Prob. that an element has a $QE \leq 9.243 \text{ m}^2$ is 0.75,
 - Prob. that an element has a $QE \leq 14.065 \text{ m}^2$ is 0.90.
- In consequence, under \mathbb{H}_0 the sampling statistics follow a multinomial distribution with parameters (N, 0.75, 0.15, 0.10).



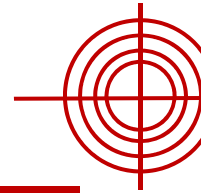
Application example

Case A

<i>Element</i>	<i>e_x [m]</i>	<i>e_y [m]</i>	<i>e_z [m]</i>	<i>QE [m²]</i>
1	-0.371	-1.672	2.755	10.524
2	-3.359	-0.815	1.454	14.063
3	0.251	0.340	0.467	0.397
4	-0.308	-0.324	0.718	0.715
5	1.172	-0.320	-0.411	1.644
6	-0.206	-3.074	-1.651	12.220
7	3.873	0.304	2.280	20.288
8	0.394	-0.442	0.989	1.330
9	2.322	-1.667	0.623	8.557
10	-1.380	-2.260	0.342	7.128
11	1.384	-1.444	-1.730	6.993
12	-1.131	-0.549	-0.930	2.447
13	0.423	0.627	-1.257	2.153
14	1.494	-1.359	-2.168	8.780
15	-1.740	0.017	-1.281	4.667
16	-1.397	-0.196	0.214	2.035
17	1.670	0.262	-2.015	6.918
18	-0.399	-1.194	1.553	3.997
19	-0.309	-1.106	1.562	3.760
20	-1.329	0.014	2.745	9.304

$$T=(15, 4, 1)$$

$$p = 0.3828$$



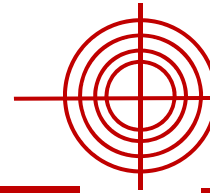
Application example

Case B

$$T = (7, 2, 11)$$

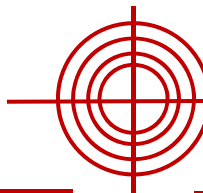
$$p = 0.0409$$

Element	ex [m]	ey [m]	ez [m]	QE [m2]
1	4.263	2.439	3.298	35.000
2	-2.547	-0.959	-0.483	7.639
3	0.876	3.985	-0.851	17.370
4	-0.010	0.352	2.098	4.526
5	-1.920	0.744	4.166	21.597
6	2.313	-1.372	-3.335	18.354
7	-2.584	2.832	0.049	14.701
8	0.830	-0.932	-0.510	1.819
9	1.895	-0.902	-2.230	9.378
10	2.242	-1.206	1.741	9.513
11	-1.341	1.723	-0.745	5.323
12	-1.457	-1.699	-4.995	29.955
13	-0.541	4.164	1.924	21.330
14	2.818	2.699	-0.834	15.920
15	0.772	-0.099	-2.907	9.059
16	3.217	-1.191	1.744	14.810
17	0.343	-0.024	4.905	24.174
18	-4.844	0.044	0.493	23.711
19	-0.050	-0.657	0.206	0.476
20	-1.096	-1.909	-1.731	7.843

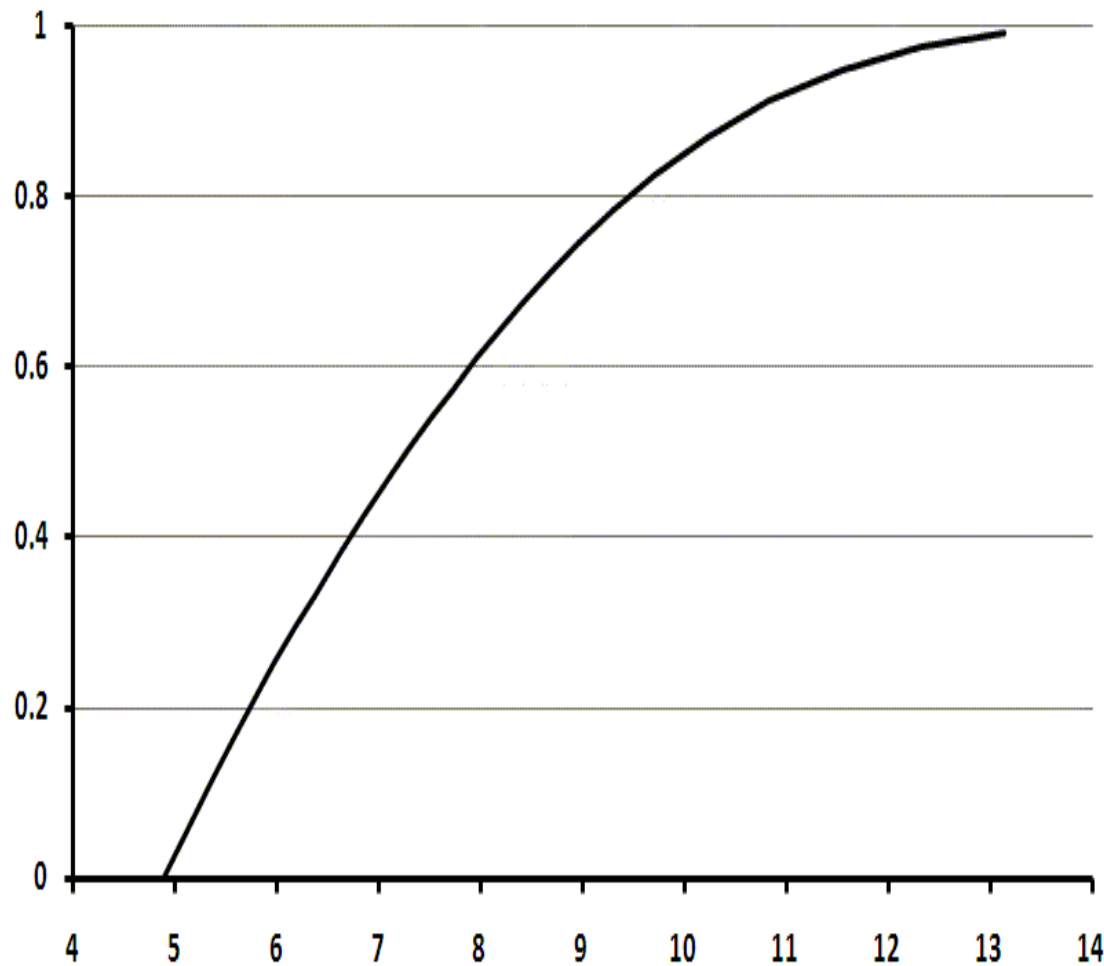
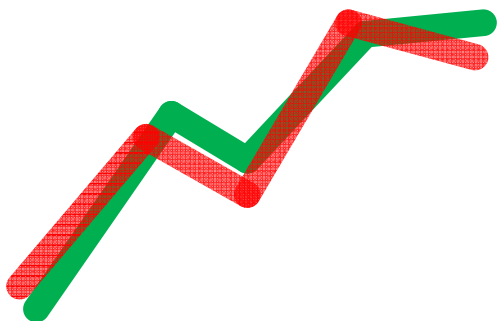
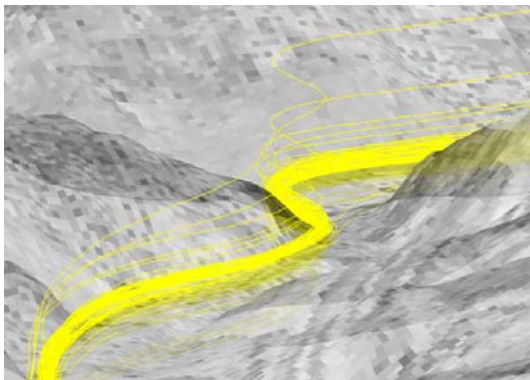


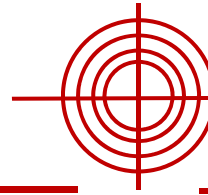
Conclusions

- We propose a general positional accuracy control method, splitting points into three categories in respect with measurement errors.
- It may be employed without any previous hypothesis about the underlying distribution of errors, and it can be applied to 1D, 2D or 3D error data.
- It is an exact statistical hypothesis testing based on multinomial distribution with three parameters, N, π_1, π_2
- The proposed statistical test is exact, so the p-value can be derived by exploring a space of solutions and summing up the probabilities of each isolated case of this space.
- This contrast has some advantages, such as its easy realization and implementation, for instance in R program.
- Albeit we have applied it in a continuous underlying situation (measurement errors), it is also useful for free distribution error models as well as discrete or categorical situations.



Conclusions

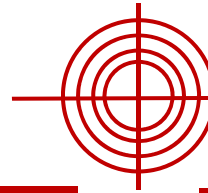




A proposal of a statistical test to control positional accuracy by means of 2 tolerances simultaneously

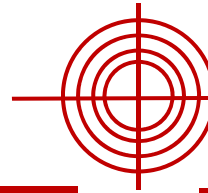
Thanks for your attention





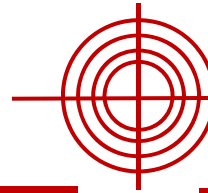
Simulation procedure

	Critical value	Sampling size (n)						
		20	50	75	100	150	200	300
Case $\pi_0 = (0.7; 0.2; 0.1)$	0.5	0.5015	0.5020	0.4980	0.5013	0.5160	0.5063	0.5002
	0.3	0.3493	0.3041	0.3003	0.2985	0.3109	0.3007	0.2998
	0.2	0.2051	0.2140	0.2001	0.2037	0.2081	0.2041	0.2006
	0.1	0.1032	0.1007	0.1013	0.0990	0.1032	0.1029	0.1035
	0.05	0.0502	0.0511	0.0506	0.0528	0.0497	0.0520	0.0514
	0.01	0.0115	0.0107	0.0105	0.0101	0.0105	0.0120	0.0111



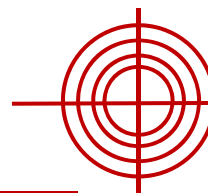
Simulation procedure

Case	Critical value	Sampling size (n)						
		20	50	75	100	150	200	300
$\pi_0 = (0.5; 0.3; 0.2)$	0.5	0.5193	0.5044	0.4926	0.5071	0.5170	0.4981	0.5028
	0.3	0.3346	0.3096	0.2981	0.3084	0.3183	0.2953	0.2944
	0.2	0.2127	0.2107	0.1974	0.2038	0.2080	0.2005	0.1968
	0.1	0.1091	0.1029	0.1026	0.1028	0.1018	0.1003	0.1045
	0.05	0.0546	0.0547	0.0529	0.0533	0.0494	0.0521	0.0507
	0.01	0.0136	0.0117	0.0103	0.0122	0.0099	0.0102	0.0094



Simulation procedure

	Critical value	Sampling size (n)						
		20	50	75	100	150	200	300
Case $\pi_0 = (0.9; 0.05; 0.05)$	0.5	0.5210	0.5205	0.5340	0.5255	0.4865	0.5035	0.4980
	0.3	0.3220	0.3260	0.3260	0.2970	0.3000	0.3015	0.3095
	0.2	0.2080	0.2145	0.1960	0.2090	0.2130	0.2060	0.2030
	0.1	0.1035	0.1185	0.1165	0.1140	0.0985	0.0960	0.1035
	0.05	0.0545	0.0570	0.0555	0.0580	0.0515	0.0520	0.0535
	0.01	0.0102	0.0115	0.0115	0.0980	0.0125	0.0125	0.0095



Simulation procedure

Proportion of times where the null hypothesis is rejected (case $\pi_0 = (0.7; 0.2; 0.1)$)

Critical value	Sampling size (n)						
	20	50	75	100	150	200	300
	True population probability: $\pi^1 = (0.65; 0.25; 0.1)$						
0.5	0.6780	0.7685	0.8195	0.8305	0.9010	0.9255	0.9610
0.3	0.5150	0.5795	0.6570	0.7000	0.7635	0.8330	0.8975
0.2	0.3645	0.4755	0.5235	0.5725	0.6620	0.7420	0.8310
0.1	0.2135	0.2985	0.3535	0.4090	0.4905	0.5870	0.7070
0.05	0.1260	0.1880	0.2315	0.2770	0.3730	0.4270	0.5615
0.01	0.0320	0.0520	0.0775	0.1015	0.1575	0.2050	0.2980
True population probability: $\pi^1 = (0.60; 0.25; 0.15)$							
0.5	0.8440	0.9305	0.9675	0.9785	0.9960	0.9980	1.0000
0.3	0.7180	0.8345	0.9060	0.9430	0.9845	0.9930	0.9990
0.2	0.5530	0.7505	0.8415	0.8995	0.9685	0.9845	0.9975
0.1	0.3935	0.5960	0.7155	0.8010	0.9150	0.9630	0.9945
0.05	0.2565	0.4560	0.6020	0.6840	0.8400	0.9205	0.9780
0.01	0.1050	0.2315	0.3240	0.4395	0.6235	0.7690	0.9095
True population probability: $\pi^1 = (0.55; 0.30; 0.15)$							
0.5	0.9140	0.9790	0.9935	0.9985	0.9995	1.0000	1.0000
0.3	0.8415	0.9500	0.9790	0.9955	0.9985	1.0000	1.0000
0.2	0.7230	0.9220	0.9620	0.9875	0.9975	1.0000	1.0000
0.1	0.5685	0.8155	0.9215	0.9660	0.9945	0.9995	1.0000
0.05	0.4240	0.7220	0.8505	0.9270	0.9855	0.9975	1.0000
0.01	0.1825	0.4540	0.6440	0.7975	0.9315	0.9850	0.9985



Simulation procedure

Proportion of times where the null hypothesis is rejected (case $\pi_0 = (0.5; 0.3; 0.2)$)

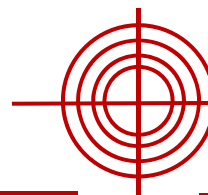
Critical value	Sampling size (n)						
	20	50	75	100	150	200	300
	True population probability: $\pi^1 = (0.45, 0.3, 0.25)$						
0.5	0.7075	0.7910	0.7970	0.8540	0.8980	0.9310	0.9620
0.3	0.5260	0.6140	0.6530	0.6755	0.7710	0.8305	0.8915
0.2	0.3800	0.5060	0.5345	0.5760	0.6635	0.7365	0.8280
0.1	0.2085	0.3065	0.3635	0.4120	0.4900	0.5860	0.7020
0.05	0.1155	0.2100	0.2280	0.2705	0.3435	0.4285	0.5635
0.01	0.0395	0.0685	0.0695	0.0985	0.1390	0.2135	0.2945
True population probability: $\pi^1 = (0.40, 0.35, 0.25)$							
0.5	0.8275	0.9240	0.9605	0.9785	0.9935	0.9965	0.9995
0.3	0.6885	0.8110	0.8860	0.9320	0.9725	0.9880	0.9980
0.2	0.5750	0.7340	0.8210	0.8865	0.9525	0.9750	0.9970
0.1	0.3740	0.5615	0.6800	0.7780	0.8885	0.9410	0.9885
0.05	0.2515	0.4400	0.5420	0.6600	0.8050	0.8910	0.9655
0.01	0.0950	0.2035	0.2770	0.3885	0.5430	0.7120	0.8795
True population probability: $\pi^1 = (0.35, 0.35, 0.30)$							
0.5	0.9315	0.9870	0.9945	1.0000	1.0000	1.0000	1.0000
0.3	0.8390	0.9540	0.9815	0.9955	0.9995	1.0000	1.0000
0.2	0.7505	0.9145	0.9685	0.9880	0.9985	1.0000	1.0000
0.1	0.5760	0.8060	0.9255	0.9615	0.9910	0.9995	1.0000
0.05	0.4115	0.7165	0.8515	0.9215	0.9780	0.9970	1.0000
0.01	0.1835	0.4640	0.6440	0.7535	0.9180	0.9865	0.9980



Simulation procedure

Proportion of times where the null hypothesis is rejected (case $\pi_0 = (0.9; 0.05; 0.05)$)

Critical value	Sampling size (n)						
	20	50	75	100	150	200	300
True population probability: $\pi^1 = (0.85, 0.10, 0.05)$							
0.5	0.8975	0.9770	0.9910	0.9965	0.9995	1.0000	1.0000
0.3	0.7915	0.9345	0.9650	0.9885	0.9965	1.0000	1.0000
0.2	0.6905	0.8740	0.9425	0.9795	0.9955	1.0000	1.0000
0.1	0.5265	0.7870	0.8805	0.9465	0.9845	0.9970	1.0000
0.05	0.4440	0.6855	0.8040	0.9025	0.9680	0.9915	1.0000
0.01	0.1905	0.4290	0.5780	0.7390	0.8960	0.9560	0.9945
True population probability: $\pi^1 = (0.80, 0.10, 0.10)$							
0.5	0.9190	0.9825	0.9940	0.9960	1.0000	1.0000	1.0000
0.3	0.7850	0.9535	0.9795	0.9855	0.9990	0.9995	1.0000
0.2	0.7560	0.8995	0.9530	0.9735	0.9960	0.9990	1.0000
0.1	0.5740	0.8085	0.9115	0.9540	0.9900	0.9955	1.0000
0.05	0.5255	0.6940	0.8430	0.9195	0.9770	0.9920	1.0000
0.01	0.1975	0.4895	0.6435	0.7210	0.9140	0.9690	0.9960
True population probability: $\pi^1 = (0.75, 0.05, 0.20)$							
0.5	0.9745	0.9980	1.0000	1.0000	1.0000	1.0000	1.0000
0.3	0.9170	0.9925	0.9980	1.0000	1.0000	1.0000	1.0000
0.2	0.9075	0.9830	0.9950	1.0000	1.0000	1.0000	1.0000
0.1	0.7855	0.9600	0.9905	0.9980	0.9990	1.0000	1.0000
0.05	0.7610	0.9120	0.9780	0.9960	0.9985	1.0000	1.0000
0.01	0.3895	0.8140	0.9255	0.9610	0.9975	1.0000	1.0000



Application example

Case C

$$\mathbf{T} = (16, 1, 3)$$

$$p = 0.5851$$

Element	e_x [m]	e_y [m]	e_z [m]	QE [m ²]
1	0.745	-0.001	-0.892	1.351
2	-1.174	-0.299	0.527	1.745
3	0.938	0.031	0.993	1.868
4	0.219	-1.092	0.651	1.665
5	-1.533	-2.152	-1.834	10.348
6	0.481	-0.010	0.497	0.478
7	-1.551	-0.163	0.902	3.244
8	-0.383	0.239	-1.118	1.453
9	-1.267	2.032	-0.887	6.519
10	1.555	2.436	-0.998	9.345
11	-0.371	-0.219	1.323	1.935
12	-0.217	0.438	0.003	0.239
13	1.606	-1.278	-0.310	4.309
14	-1.338	-0.733	0.132	2.345
15	-0.365	1.711	0.526	3.336
16	-1.115	-1.208	-0.971	3.643
17	0.004	-0.203	0.307	0.135
18	-1.031	0.998	-0.232	2.114
19	0.740	-0.638	-0.397	1.114
20	0.861	-0.080	0.879	1.521