Variability of NSSDA Estimations

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Abstract: Using a statistical simulation process the variability of National standard for spatial data accuracy (NSSDA) estimations are analyzed according to sample size. Simulation results show: (1) that the NSSDA positional accuracy estimation has a variability of 11% when using the minimum recommended sample size of 20 points; and (2) that the use of samples of 100 points is needed in order to reach an effective confidence level of 95%. The NSSDA is a methodology of shared risk between users and producers when accuracy is "as expected," but for other cases the relation is altered. As simulation results demonstrated, this change is depicted by means of a family of acceptance curves that can be used by users to determine the sample size for limiting their acceptance risk, but also by producers to analyze their rejection risk.

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Introduction

The positional accuracy of cartographic products has always been of great importance. It is, together with logical consistency, the quality element of geographic information most extensively used and evaluated by the national mapping agencies (Jakobsson and Vauglin 2002). In a geographic data base (GDB) the position of a real world entity is described with values in an appropriate coordinate system.

Positional accuracy represents the nearness of those values to the entity's "true" position in that system. The positional accuracy requirements for a GDB are directly related to its intended use(s). Positional accuracy is determined by means of a statistical evaluation of random and systematic errors (DOD 1990) and specified by means of the root mean squared error (RMSE) or by the mean value of errors (μ) and their standard deviation (σ). Comparison with an independent source of higher accuracy is the preferred method for assessing positional accuracy (ANSI 1998).

Since positional accuracy is essential in cartographic production, mapping agencies have used statistical methods for its control. In the United States (USA) there are several recognized standard methodologies which can be used for specifying spatial data products, and resultant positional accuracy compliance criteria, such as: the national map accuracy standard (USBB 1947), the accuracy standards for large scale maps (ASPRS 1989), the engineering map accuracy standard (ASCE 1983), or the more recent national standard for spatial data accuracy (NSSDA) (FGDC 1998).

The NSSDA is a statistical methodology and a compulsorily fulfilled standard for federal agencies of the United States producing analogical and/or digital cartographic data. Despite its very recent adoption in 1998, the NSSDA has achieved a great impact not only in the United States but all over the world because the United States and its regulations always have great impact and perform an international leadership.

As in other positional quality control procedures, in the NSSDA the coordinates of a set of points in the GDB are compared to coordinates of the same points in a higher accuracy source, generally a field survey. In this way a RMSE is derived from discrepancies between pairs of coordinates. The NSSDA does not carry out a study on the presence of bias, as it considers that "they might have been eliminated in the best way" (FGDC 1998). Therefore, the NSSDA only focuses on the study of data dispersion. The NSSDA gives results in a more open way than the previous methodologies because it leaves it up to the user whether or not the derived accuracy reaches expectations, which means, in a practical way, if the product passes or fails the user's accuracy expectations. So acceptance or rejection is the responsibility of the user. The test only tells us: "the product has been checked/compiled for N meters of horizontal/vertical accuracy at 95% of level of confidence."

From a statistical point of view, one of the most controversial aspects of all the methodologies for positional control is the number and distribution of the control points. With regard to the number, which is our interest here, it should always be large enough for the hypothesis of normality to be fulfilled, this being determined by the laws of large numbers in statistics. For this reason recommendations always suggest at least 20 points (FGDC 1998; MPLMIC 1999). Obviously, since an elimination of gross errors should always be performed, a higher number should be captured. Nevertheless, light of other works (Li 1991) and the control processes of other institutions (Newby 1992; NJUG 1988) this size ($n=20$) seems to be very small and larger sizes are suggested (Atkinson 2005). The number of points should be enough to ensure, with a given level of confidence, that a GDB with an unacceptable quality level will not be acquired. On the other hand, the number of points to be used for the control must be the lowest

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possible in order to minimize the cost of such a control.

As an example of responsibility and leadership there is a current proposal for the revision (FGDC 2003) of the NSSDA in order to improve and explain its technical definition, and also to fit its use to some users and producers' demands. So the proposal is based on various suggestions like those coming from the National Digital Elevation Program (NDEP 2006) oriented towards adding instructions for how to test and report vertical accuracy in areas where a normal distribution cannot always be attained, or the proposal of Tilley (2002) for classifying accuracy results derived from the NSSDA, or the claim by McCollum (2003) that the Greenwalt and Shultz (1962) estimator is inappropriately used in the NSSDA to determine a probability of 95%.

From our point of view, the NSSDA revision is a good opportunity to improve, along with those mentioned above, other aspects that also need some attention:

1. Give information on the statistical behavior of the methodology;
2. Related to the previous, give clear and better recommendations about the number, kind, and distribution of control points to be used and their influence on the variability and reliability of results; and
3. Adding instructions and advice on how to deal with bias.

The behavior of a methodology should be known for its appropriate application. The NSSDA is technically robust (Tilley 2002) but the standard does not give any information about its behavior. Our work is a contribution centered on the statistical behavior of the NSSDA. We have faced what we considered to be the major problem when applying the NSSDA: its variability. By means of a simulation methodology we have analyzed the interdependence between NSSDA estimations and the sample size of points used for the control.

This paper is organized in four sections: The first presents the simulation methodology used to develop the research; the second section deals with the analysis of the accuracy variability depending on sample size but with fixed population variability, and the next presents the same analysis when variability of populations is considered. Finally, conclusions are presented.

Simulation Methodology

Simulation has been used as the base tool for analyzing the behavior of the NSSDA methodology in the planimetric case. For the simulation we applied the Monte Carlo method which requires a large amount of random numbers. Simulation is an easy method to understand because it is based on the reproduction of a known process or system, so that theoretical aspects are obviated. Simulation can be defined as the construction of a mathematical model to reproduce the characteristics of a phenomenon, system, or process, using a computer, in order to obtain information or solve problems (Ríos et al. 1997). It is a very interesting tool (Wikipedia 2007) for many fields such as engineering, training, medicine, intelligence, etc. With simulation a decision maker can try out new designs, layouts, processes, and systems before committing resources to their acquisition or implementation; gain insight into which variables are most important to performance and how these variables interact; better understand how the system really operates; and compare alternatives and reduce decision risk (Ariza 2002).

There is no single prescribed methodology under which simulation studies are conducted. Most simulation studies follow four major steps (Ríos et al. 1997): formulating the problem, developing the model, running the model, and analyzing the output.

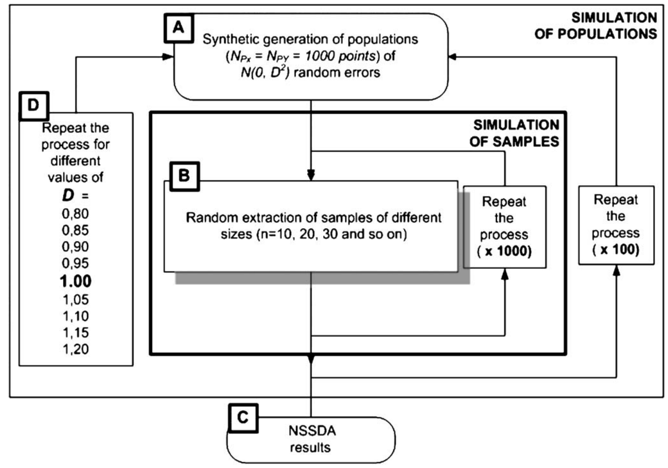

Fig. 1. Simulation process flow

Our research focuses on the study of the variability of the estimated positional accuracy, but only horizontal, of a GDB when applying the NSSDA to different sample sizes. So we are going to replicate the NSSDA application process under controlled circumstances by means of synthetic populations of errors. Here some basic assumptions are made:

1. In order to enable the generalization of results, synthetic normal ($\mu_P=0$, $\sigma_P^2=1$) distributed populations of data are used as if they were positional errors derived from a control survey of higher accuracy. Note that: (1) it is considered that NSSDA controls are applied to GDBs with no bias ($\mu_P=0$); and (2) μ and σ units of measure are in meters; and
2. Because of the simulation it is possible to imagine that the spatial distribution of control points is always adequate, not affecting the validity of results.

The developed model, or the simulation process, basically consists of the following (Fig. 1):

1. Simulation of populations (A): populations of well known parameters ($\mu_P=0$, $\sigma_P^2=1$) are derived from a controlled statistical random values generation process. Single population values are considered positional error values. Here a total of NR_{SP} synthetic populations will be realized; and
2. Simulation of samples (B): samples of different sizes ($n=10$, 20, 30, etc.) are randomly extracted from each population. The NSSDA is applied to each sample as if it were a single positional control test. For each sample size NR_{SAM} samples will be extracted for each population.

The Monte Carlo method is conceptually related to sampling, so the number of times or runs NR a process must be iterated can be derived from sampling recommendations. Heuvelink (1998) presents a discussion on NR. Here $NR_{SP}=100$ and $NR_{SAM}=1{,}000$; so the total number of runs considered for each sample size is $NR=NR_{SP}\times NR_{SAM}=100{,}000$; this is in order to obtain statistically sounder results with the lesser variations of estimated deviations (stability) and because the numerical load is not a problem here.

The output of the process is analyzed from the statistical computations (C in Fig. 1). Because the simulation is performed using a normal $N(\mu_P=0,\sigma_P^2=1)$ distributed population, the theoretical value to be estimated by the NSSDA is $\text{ACCURACY}_{R(\text{theoretical})}=2.447$ m, which corresponds to a circular error estimation with a probability of 95%. For each sample size ($n=10,20,30$, etc.) resulting values of NSSDA accuracies are aggregated, deriving means, deviations (variability), and the variations of deviations (stability). For the analysis performed in the section "Variability of Acceptance of NSSDA Accuracy Estimations" we also considered the possibility of different population deviations, meaning different variational behaviors. In this case the simulation of populations (synthetic generation of populations) is repeated for different values of variance (labeled D in Fig. 1).

Variability of NSSDA Accuracy Estimations

The result of an estimation process can be expressed as a mean value for the statistical estimator and its deviation, or variability from the mean value. Both mean and deviation are affected by sample size, but especially deviation. High variability of an estimator means that the estimation is not fine, and for a given case it can take values which could create problems through the loss of accuracy incurred. In this way we can express the actual estimation as a function of the estimator and its variability in the following form

$$\text{ACCURACY}_{R(\text{actual})} = \text{ACCURACY}_{R(\text{estimator})} \pm K \text{ Deviation(m)} \quad (1)$$

where K=coefficient taking into account an assumed confidence level upon the deviation of the estimator (for instance, $K=1.96$ for a confidence level of 95% on the assumption of normality distribution for deviations).

The results of the simulation process are shown in Table 1 (here $K=1$). The second column shows the estimator value, the next its deviation or variability expressed in the same units as the estimator, and the following in percentage. The last column is an index of the quality of the process. Because of the large number of simulations the final results are very sound. For worst cases, samples of minimum size, stability values are better than 1%, which means a very high coherence within the results.

As can be observed in Table 1, for the minimum size recommend by the NSSDA ($n=20$) the observed value obtained is $\text{ACCURACY}_{R(\text{actual})}=2.432\pm0.270$ m. Here the mean value is 0.5% less than the corresponding theoretical value to be detected, and variability is on the order of ±11%. To limit the variability to a maximum of $5\% (=\pm2.5\%)$ a sample with at least $n=275$ points will be needed. The variation range decreases when sample size increases, so that for $n=700$ points it is $\sim1\%$, and the mean value is very close to the supposed one.

Until now we have assumed $K=1$, but we can also consider a confidence level for the deviations, for instance 95% which means that $K=1.96$. So if we want an estimation with a precision of, say, 0.2 m for the 95% confidence level, this implies a deviation of $0.2/1.96=0.1$ m, and entering this value in Table 1 (the third column) the suggested sample size is $n\approx125$ (first column).

Another important factor to be considered is that deviations of the estimator affect the actual confidence level of the NSSDA, modifying its theoretical value of 95%. For example, in the case where $n=20$ the estimation gives the actual possibility of obtaining values in the interval $\text{ACCURACY}_{R(\text{actual})} \in [2.162; 2.702]$ which implies approximate confidence levels, derived from a circular normal distribution, in the range of 90–97%.

The same results of Table 1 are expressed graphically in Fig. 2. Here the X axis refers to the size of the control sample, and the Y axis to the mean observed, or estimated, population value through the sample when using $N(\mu_P=0,\sigma_P^2=1)$ populations in the simulation process. The wider and dashed-dot line corresponds to the value to be theoretically detected by the NSSDA ($\text{ACCURACY}_{R(\text{theoretical})}$). The series of points are the results of the simulation; they have a very clear tendency, approaching the theoretical value from below when the sample size is increased. The other two dashed lines represent the decreasing tendency of the variability.

As shown in Fig. 3 (a zoom detail of Fig. 2), the mean estimated value for NSSDA control is a little lower than the corresponding value for the $N(\mu_P=0,\sigma_P^2=1)$ distributed population. In other words, in mean values the NSSDA underestimated the error level of the population, or overestimated the accuracy. Nevertheless such underestimation is very small, and also presents a tendency to be null. Numerically speaking the differences range from 2.1% for $n=10$ up to 0% for $n=800$.

Table 1. Mean ACCURACY_R Values and Variability Obtained by Simulation of Samples and Populations

Sample size n[a]	ACCURACY_R (m)[b]	Deviation $\pm m$[c]	Variation $(\pm\%)$[d]	Stability (%)[e]
10	2.416	0.382	15.8	0.7
15	2.426	0.312	12.9	0.6
20	2.432	0.270	11.1	0.5
25	2.434	0.241	9.9	0.5
30	2.437	0.219	9.0	0.5
35	2.438	0.203	8.3	0.4
40	2.439	0.189	7.8	0.4
45	2.440	0.178	7.3	0.4
50	2.441	0.168	6.9	0.4
55	2.441	0.160	6.6	0.4
60	2.442	0.153	6.3	0.4
65	2.442	0.147	6.0	0.4
70	2.442	0.141	5.8	0.4
75	2.443	0.136	5.5	0.4
80	2.443	0.131	5.4	0.4
85	2.443	0.127	5.2	0.4
90	2.443	0.124	5.1	0.4
95	2.444	0.121	5.0	0.4
100	2.444	0.115	4.7	0.4
150	2.446	0.088	3.6	0.3
200	2.445	0.075	3.1	0.4
250	2.445	0.065	2.6	0.4
300	2.446	0.058	2.4	0.3
350	2.446	0.052	2.1	0.3
400	2.446	0.046	1.9	0.3
450	2.446	0.042	1.7	0.3
500	2.446	0.039	1.6	0.3
600	2.446	0.031	1.3	0.2
700	2.446	0.025	1.0	0.2
800	2.447	0.024	1.0	0.2

[a]Size (number of points) of the 1,000 random samples.

[b]Simulation mean observed value for horizontal accuracy (ACCURACY_R) by applying the National Standard for Spatial Data Accuracy with 95% confidence level.

[c]Mean deviation of the simulation process with respect to the mean observed value.

[d]Previous deviation expressed as a percentage of mean observed value of the horizontal accuracy.

[e]Stability of the process when using 100 random populations, distributed as normal ($\mu=0$, $\sigma^2=1$).

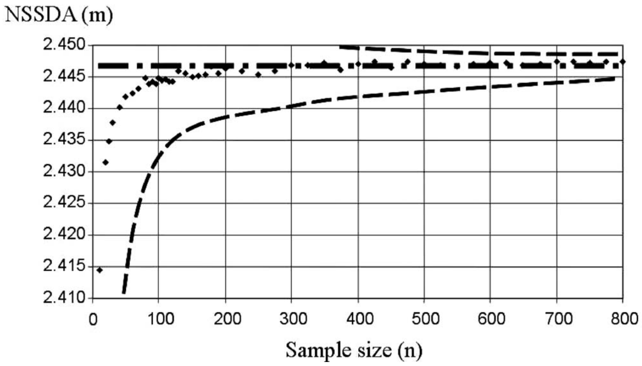

Fig. 2. Mean ACCURACY$_R$ values (points) and variability (dashed lines) obtained by simulation. Theoretical National Standard for Spatial Data Accuracy value corresponding to $N(\mu_P=0,\sigma^2 P=1)$ distributed population is shown by wider and dashed-dot line.

Variability of Acceptance of NSSDA Accuracy Estimations

Until now we have worked under the assumption of having a $N(\mu_P=0,\sigma_P^2=1)$ distributed population and analyzed what can occur when estimating from the sample of a given size n. Now we are going to analyze the behavior of the NSSDA when expecting $N(\mu_P=0,\sigma_P^2=1)$ distributed population but actually working with other normal $N(\mu_P=0,\sigma_P^2=D^2)$ distributed population where $D\neq 1$. In this way we can determine the variability of the acceptance of the NSSDA estimations, an analysis which allows us to derive the acceptance curves for the same. In other words the question is: if I control a batch of products expecting a NSSDA accuracy of 2.447 m (ACCURACY$_R$), how many times (in a mean percentage value) will I get an accuracy of 2.447 m?. It is obvious that this depends on the variability of the products themselves, which has great importance for both producers and users.

For this analysis we have used a simulation process similar to that mentioned above, but changing the variation behavior when creating random populations. So a set of populations normally distributed has been synthetically created following a $N(\mu_P=0,\sigma_P^2=D^2)$, where $D=0.8$, 0.85, 0.9, 0.95, 1.00, 1.05, 1.10, 1.15, and 1.20. For each synthetic population 1,000 samples of different sizes ($n=10,20,30$, etc.) were extracted. The NSSDA was applied to each sample as if it were a single positional control test.

The different values of D can be considered, in relation to $D=1$ m, as variability ratios implying a detected nominal accuracy value when applying the NSSDA, and vice versa. This idea is presented in Table 2.

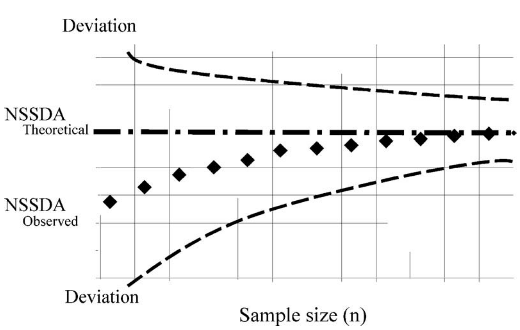

Fig. 3. Detail of Fig. 2

Table 2. Variability Ratios and Corresponding Detected Nominal Values for National Standard for Spatial Data Accuracy (ACCURACY$_R$) for Different Population Deviations

Population deviation (D)	Variability ratios (%)	ACCURACY$_R$ at 95% (m)
0.80	−20	1.957
0.85	−15	2.079
0.90	−10	2.201
0.95	−5	2.324
1.00	—	**2.446**
1.05	+5	2.568
1.10	+10	2.691
1.15	+15	2.813
1.20	+20	2.935

In order to understand the acceptance index in the form of probability (percentage) we consider the following rules:

1. If $D<1$, the accuracy of the population is better than expected, and this means that it would be considered satisfactory or accepted. So when performing the simulation, we will take into account the number of cases where observed resulting values for ACCURACY$_R<2.447$ m. The number of such cases will be expressed as an acceptance percentage of total cases (the number of times we are able to determine that accuracy is better than expected); and
2. If $D>1$, the accuracy of the population is worse than expected, and this means that it would be considered not satisfactory or not accepted. So when performing the simulation, we will take into account the number of cases where observed resulting values for ACCURACY$_R<2.447$ m. The number of such cases will be expressed as an acceptance percentage of total cases (the number of times we are not able to determine that accuracy is worse than expected).

The results of this process are presented in Fig. 4, which shows a family of curves we call acceptance curves. The horizontal axis corresponds to sample size (n) and the vertical to percentage of acceptance (%). The black wider curve in the middle, corresponds to the case where $D=1$, a previously studied situation where population follows a $N(\mu_P=0,\sigma_P^2=1)$. Here it can be observed that this curve is somewhat above 50%. This is because the NSSDA underestimation implies a similar increment of the per-

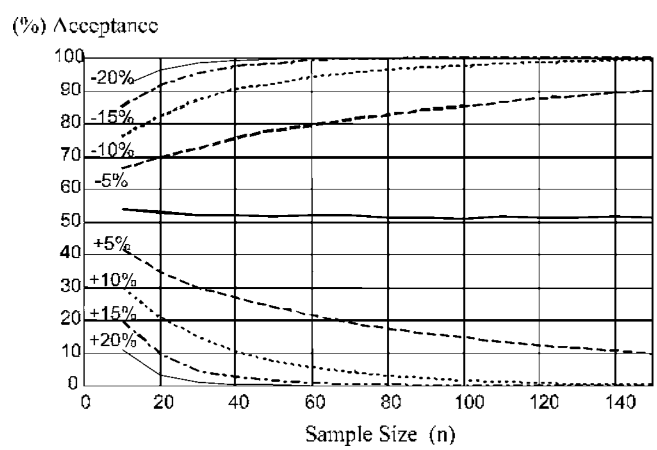

Fig. 4. Evolution of acceptance levels for different variability in population deviations versus sample size

centage of acceptance. Curves above the wider line correspond to those cases were $D<1$, labeled −5% up to −20%, and curves below to those where $D>1$, labeled +5% up to +20%.

When $D<1$ quality is better than expected (lesser variability) user acceptance increases when sample size n also increases. Here there is a risk for the producer because the product is good enough in variability but is not accepted in a percentage that equals 100% minus acceptance (%) (a good product can be rejected in this percentage). On the other hand when $D>1$ quality is worse than expected (greater variability), user acceptance is at risk (a bad product can be accepted in this percentage), but the risk decreases when sample size n increases. Fig. 4 also shows that however lesser or greater the deviations are in relation to the expected, the smaller sample size is needed for a given acceptance or rejection risk.

An example can help us to understand the above. Let us consider a batch of products to which the NSSDA is applied. If the positional error of the product follows a $N(\mu=0,\sigma^2=0.95^2)$ and we expect a behavior of a $N(\mu=0,\sigma^2=1^2)$, variability of this data is 5% less than expected (it means it is 5% better) and the acceptance follows the curve of Fig. 4 labeled −5%. For this situation, if the sample size of control points is $n=60$, the percentage of acceptations in the batch is on average on the order of 80%, which also means a risk of 20% for the producer, because the product is actually better than expected. In industrial processes the producer's and user's risk are commonly limited to 5 or 10%, respectively. So unless the difference between the actual and expected variability is greater than 15%, the sample size provides the best protection for the interests of both producer and user. This can be seen from a different point of view: the interest for the producer to create GDBs with positional accuracies at least on the order of 15% better than the stated threshold for the assessment, and thus to reduce his rejection risk, which is the same as increasing the capacity of his production process. This also allows a reduction of the sample size of the positional accuracy assessment. So there is a need for a agreement between the user (acquirer) and the producer in order to decide where to direct the resources: to control or production. For particular cases the answer comes from the cost functions of both processes. In general, the best is quality assurance in production and reduction of controls.

Conclusions

The behavior of a methodology should be known for its appropriate application. The NSSDA is technically robust but the standard does not give any information about its variability. By using a simulation based methodology the NSSDA ACCURACY_R estimation variability has been analyzed. The statistical analysis is based on the use of normal distributed synthetic populations, which ensures the control of the process, the generality of results, and their easy applicability to real cases. The main conclusions derived from our research are as follows:

1. The NSSDA has a small tendency to underestimate accuracy;
2. For the minimum proposed sample size ($n=20$ points) the variability of results is on the order of ±11%, which actually means a variability of the confidence level in the approximate range of 90–97%;
3. In order to obtain 95% confidence level on estimation and variability within a range of ±5%, the sample size must be on the order of 100 points;
4. If the variability of the population is greater or lesser than expected the behavior of the NSSDA results is depicted by a family of acceptance curves; and
5. Acceptance curves can be used by users to determine the sample size for limiting their acceptance risk, but also by producers to analyze the tradeoffs between their product's quality and acceptance ratios in order to decide and establish the capacity of the production process.

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