Acceptance Curves for the Positional Control of Geographic Databases

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Abstract: A simulation process is implemented in order to generate synthetic populations of positional-planimetric errors and later, by a bootstrap process, a family of acceptance curves (nomogram) has been derived. These curves allow us to learn the acceptance level, or pass level, of a geographic database when a positional control test based on bias and on variability is applied to it against a given standard. Acceptance curves allow us to report user's risk and to determine the adequate size of the sample in order to reduce this risk to a desired level.

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Introduction

The positional accuracy of cartographic products has always been of great importance. It is, together with logical consistency, the quality element (ISO 2002) of geographic information most extensively used and evaluated by the national mapping agencies (Jakobsson and Vauglin 2002). Good positional quality is needed when using two different geographic databases. Different positional behaviors of geographic data sets mean the existence of interproduct positional distortion and a barrier to interoperation (Church et al. 1998). This barrier is not only positional and geometrical but also thematic, being greatly affected by position (McGwire 1996; Carmel et al. 2001, 2006). In the information society where global positioning system (GPS) devices are broadly used by citizens in conjunction with digital cartographic products, everybody is able to obtain coordinates, and in some way to control or test cartographic products. Thus, positional accuracy is now a paramount factor for mapping agencies.

Positional quality is one of the components of geographic data (Morrison 1995), and is determined by positional accuracy. Quality can be defined as "fitness for use" (Juran and Gryna 1970), so from the positional perspective several definitions can be established in accordance with user's interests (DOD 1990): absolute and/or relative accuracy, horizontal/vertical accuracy, and so on; but the statistical aspect is present in all of them.

Given that positional quality is essential in cartographic production, all mapping agencies have used statistical methods for its control. These controls are applied to geographic databases (GDBs) or cartographic products, and we call them positional control tests (PCTs). Among the different methods used, we can highlight the National Map Accuracy Standard (USBB 1947), the Engineering Map Accuracy Standard (ASCE 1983), the Accuracy Standards for Large Scale Maps (ASPRS 1989), or the more recent National Standard for Spatial Data Accuracy (FGDC 1998). These PCTs can be used as a basis for specific procedures of positional quality control within a quality management system such as ISO 9001 (ISO 2000).

Since the late 19th century quality control techniques have been developed focusing on industrial activities, producing numerous methods for the statistical control of processes (e.g., lot plot, P, $\bar{X}$, and S graphics, etc.) (Hansen and Ghare 1990) (see a cartographic example in Simley 2001). According to Pyzdek (1989) quality control is the science of discovering and controlling variation. The purpose of quality control is to establish and maintain conformity of the products with design requirements, mainly expressed as standards or specifications. One of the most important aspects of statistical quality control is the acceptance control of products. In these cases, the final product is studied without taking into account the production process; it is a "black box" type analysis in which both acceptance and rejection are decided considering statistical criteria.

Industrial products are usually controlled by their attributes (ISO 1999) or by variables (ISO 2005) mostly considered as one dimensional. Cartographic positional quality control presents certain specific peculiarities which require a different treatment so that methods designed for and introduced into industries are not adequate for their direct application to positional acceptance or rejection of cartographic products.

As in industrial acceptance processes, positional control is based on sampling. PCTs can be used as tools for acceptance quality control of GDBs coming from suppliers. When acceptance sampling is performed in industry a sampling plan is required in order to establish and make explicit to both producer and acquirer (user) the conditions of the process: Definition of lot, lot size, sampling method and size, producer's and user's risk, cost, etc. (AEC 1990). All of these should be stated in the supply contract for GDB. An operating characteristic curve is an analytical or graphical expression of the statistical aspects of a sampling plan, which is also the foundation of such very important standards as

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ISO 2859-1 (ISO 1999). Despite this, references to operating characteristic curves application to spatial data quality are very scarce. Caspary and Joos (2002) make an abridged presentation with an interesting linking to cost and Ariza (2002) points out its use for positional controls. In order to understand our methodology, developed specifically for the planimetric case, both the bases of the quality control in reception and our proposed positional control methodology are presented in the next two subsections. Later we deal with the simulation methodology applied in order to derive the acceptance curves, and this mainly consists of a two step simulation process: Population simulation and positional control simulation upon subsamples. The result can be graphically expressed as a nomogram, so the results section is devoted to explaining this nomogram and an example is included in order to show how to use the nomogram as well as the benefits of applying this methodology. Finally the main conclusions are stated.

Quality Control in Reception

When buying a lot of products, the best way to decide whether or not to accept it is to check 100% of the elements. However, this is not always possible because of the cost or because, for some products, the only way to check quality is by destruction (Besterfield 1994). For this reason it is very common to use statistical sampling methods called sampling plans for random checking.

The decision from an investigated sample on whether or not a lot is satisfying stated requirements can be carried out through hypothesis testing. A statistical hypothesis is a statement about the values of the parameters of a probability distribution. In statistical testing two alternatives are always considered: H_0: so-called Null Hypothesis and H_1: so-called Alternative Hypothesis.

In the case of geodata, H_0 gives the quality M_0 a customer would expect, whereas the H_1 refers to the lowest quality M_1 a customer can accept for his particular application (Caspary and Joos 2002), but here there is a very important assumption: Data were produced by an in-control process. This means that the process has the capability to produce within the stated tolerances, and process capability is directly related to process variability (Montgomery 2001).

As sampling is a random procedure, two kinds of errors may be committed when testing hypotheses. If the null hypothesis is rejected when it is true, then a Type I error (α) has occurred. If the null hypothesis is not rejected when it is false, then a Type II error (β) has occurred. A Type I error is called producer's risk because it denotes the probability that a good lot will be rejected, or the probability that a process producing acceptable values of a particular quality characteristic will be rejected as performing unsatisfactory ones. Type II is called consumer's risk because it denotes the probability of accepting a lot of poor quality. But sometimes it is more convenient to work with the power of the test (Montgomery 2001), which is the probability of correctly rejecting H_0 (Power$=1-\beta=P\{\text{reject } H_0/H_0 \textit{ is false}\}$).

When testing statistical hypotheses the general procedure is to specify a value for α and then to design a test procedure so that a small value of β is obtained. Thus, the producer's risk is directly controlled or chosen by α and the consumer's risk is generally a function of sample size and is controlled indirectly. The larger the size of the sample, the smaller the consumer's risk (Montgomery 2001).

A sampling plan is defined by (AEC 1990): the lot size N, sample size n, sampling method, acceptance Ac, and rejection Re values. The behavior of a sampling plan can be expressed in terms of its operating characteristic curve (Caspary and Joos 2002). The operating characteristic curve is a graph which shows β as a function of the difference d between M_0, which corresponds to H_0, and the actual value M_{actual} derived from the sample (Hansen and Ghare 1990). The larger the difference, the smaller the consumer's risk. But β can also be plotted against a ratio of deviations (Montgomery 2001), e.g., the process standard deviation to the actual standard deviation derived from the sample. For a detailed discussion of the use of operating characteristic curves, refer to Hines and Montgomery (1990) or Montgomery and Runger (1999), or other general quality control manuals such as in Hansen and Ghare (1990), Besterfield (1994), Sebastián et al. (1999), and Montgomery (2001).

Positional Quality Control

All mapping agencies use statistical methods for positional accuracy assessment and control, stated generally in the form of standards, which have mainly been extended in their application to the extent of their influence area. Standards are very important because they mean technology but also an economic optimization of the quality of geographic information (Krek and Frank 1999): with a quality standard the producer provides the product according to the known specification and characteristics, as defined in the standard. This assures a certain level of reliability and certainty, allowing the buyer to avoid excessive measuring of quality and thus reducing the measuring cost and shortening the buyer's decision-making process.

A process or a production system that is under control produces an output flow that is characterized as a random or stochastic process; it is stable and predictable, so the quality of the output is known to deviate from a target value (the standard) within well defined limits (Deming 1986). The variation in a stable process is said to be due to common or chance causes and its level limited to a tolerated amount. In order to maintain a process under control it is necessary to detect and remove special, or assignable, causes of the process which engender nonrandom patterns of variation or greater patterns of variation. The main objective of statistical quality control is to determine whether variations are due to common causes or to special causes that require corrective action. In this sense, quality control is essentially concerned with variation (Schmidley 1997).

The previous idea can be more or less easily recognized in standard methodologies (e.g., USBB 1947; ASCE 1983; ASPRS 1989; FGDC 1998; and so on) used for specifying spatial data products and the resultant positional accuracy compliance criteria. From our point of view, the standard that performs best for a positional quality control of a GDB is that which has a sound statistical basis, and also gives more information about the process. This enables us detect problems and then find their root causes and eliminate them, thus maintaining the process under control. For this reason we like to be informed of the existence of systematic errors (bias) and of the variations of the process. The most appropriate statistical methodologies for detecting both behaviors are two commonly used statistical hypothesis tests (Caridad 1985): The first for bias by means of a t-student test, and the second for precision or variation by means of a chi-squared test. Each test must be applied to the X and Y planimetric components, so four tests are performed: T_X, T_Y, chi_X^2, chi_Y^2, and it is necessary to pass all of them in order to consider that the GDB is accepted. The same two tests T_Z, chi_Z^2 will be applied for the Z

(altimetry), but here we only consider the planimetric case because the Z case is a simple one. Our preference basically coincides with the proposal of the ASCE (1983) and Sevilla (1991). We have only introduced two considerations: (1) verification of basic needed hypotheses (randomness and normality) and (2) the Bonferroni corrections (Bonferroni 1935; Miller 1991) for the α levels in each test in order to attain a global significance of 5%.

Thus the proposed statistical procedure is based on two main hypotheses: Randomness and normality of positional errors; and both should be checked. The assumptions of randomness and normality are very common in statistical testing, and also when dealing with positional controls, but are rarely performed perhaps because they are easily accomplished by data, or because of the additional problems with performing two statistical tests (Atkinson 2005). Obviously, both tests should be passed before the application of bias and variability tests. Randomness means that errors follow a random behavior with independence of their distribution. Randomness can be checked through different statistical tests like the runs test, also called the Wald–Wolfowitz test (Caridad 1985). As Simley (2001) points out, Shewhart (1931) recognized that quality measurements in general fracturing were distributed normally. Despite the fact that some works, like those of Thompson and Rosenfield (1971) and of Gustafson and Loon (1982), indicate that errors are not normally distributed, many other researchers, like Mikhail (1976) and Goodchild and Gopal (1989), consider that random variables that represent measurements in cartography, photogrammetry, geodesy, or surveying are often nearly normally distributed. Normal distribution of errors is a common explicit or inexplicit assumption in many statistical models dealing with positional errors like the works of Li (1991), Shi (1998); Leung and Yan (1998), or Shi and Liu (2000). Normality of data can be verified by means of a general test such as Kolmogorov Smirnof or more specific such as D'Agostino-Pearson.

As mentioned previously, the process applied in our work has only been developed for planimetric control, and is as follows:

- Step 1: Sample size. Determine a sample of size "n" of "well distributed" and "well defined points."
- Step 2: Error estimation. Compute the difference between positions of points on the product and on the source of higher accuracy. Field units are used (e.g., meters)

$$e_{xi} = x_{\text{True } i} - x_{\text{GDB } i} \tag{1a}$$

$$e_{Yi} = y_{\text{True } i} - y_{\text{GDB } i} \tag{1b}$$

- Step 3: Hypothesis fulfillment. Compute statistical tests to verify randomness and normality of error data.
- Step 4: Mean and deviation. Compute the average error (μ_X, μ_Y) and deviation (S_X, S_Y) for each component

$$\mu_X = \frac{1}{n}\sum_{i=1}^{n} e_{xi} \tag{2a}$$

$$\mu_Y = \frac{1}{n}\sum_{i=1}^{n} e_{Yi} \tag{2b}$$

$$S_x = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} (e_{xi} - \mu_x)^2} \tag{3a}$$

$$S_Y = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} (e_{Yi} - \mu_Y)^2} \tag{3b}$$

- Step 5: Estimate Bias. Compute observed t-student $(T_{\text{OBS } X}, T_{\text{OBS } Y,})$ statistic for each component

$$T_{\text{OBS } X} = \frac{\mu_X\sqrt{n}}{S_X} \tag{4a}$$

$$T_{\text{OBS } Y} = \frac{\mu_Y\sqrt{n}}{S_Y} \tag{4b}$$

- Step 6: Estimate Variability. Compute observed chi-square $(\chi^2_{\text{OBS } X}, \chi^2_{\text{OBS } Y})$ statistic for each component

$$\chi^2_{\text{OBS } X} = \frac{S_X^2(n-1)}{\sigma_X^2} \tag{5a}$$

$$\chi^2_{\text{OBS } Y} = \frac{S_Y^2(n-1)}{\sigma_Y^2} \tag{5b}$$

- Step 7: Threshold values. Obtain from a table or appropriate analytical function the threshold values $T_{n-1,\alpha/8}$ and $\chi^2_{n-1,\alpha/4}$ for the t-student and chi-square (χ^2) distributions.
- Step 8: Pass/Fail. The GDB pass the complete test with 95% confidence if the following four tests $(T_X, T_Y, \text{chi}_X^2, \text{chi}_Y^2)$ are passed.
  For bias, the hypothesis is $H_{0,\text{bias}}: \mu = 0$.

$$\text{If}|T_{\text{OBS } X}| \leqslant T_{n-1,\alpha/8} \rightarrow \text{PASS for bias in } X$$

$$\text{If}|T_{\text{OBS } Y}| \leqslant T_{n-1,\alpha/8} \rightarrow \text{PASS for bias in } Y$$

  For variability, the hypothesis is $H_{0,\text{Variability}}: S \leqslant \sigma$.

$$\text{If}|\chi^2_{\text{OBS } X}| \leqslant \chi^2_{n-1,\alpha/4} \rightarrow \text{PASS for variability in } X$$

$$\text{If}|\chi^2_{\text{OBS } Y}| \leqslant \chi^2_{n-1,\alpha/4} \rightarrow \text{PASS for variability in } Y$$

As can be observed, Steps 1–7 are basic computations of the methodology; the statistical tests are stated in the eight step whose interpretation is as follows:

- The mean error μ is an estimation of bias in the population (or product). Our interest is that bias will be zero, in a statistical sense. So the null hypothesis is stated as: $H_{0,\text{Bias}}: \mu = 0$. If the $H_{0,\text{Bias}}$ is rejected we must look for the causes of bias and eliminate them.
- The standard deviation S of the sample is an estimation of the population (or product) variability σ. Here our interest is that sample variability will be lower, or at least equal, to the supposed variability of the population (product). The null hypothesis is stated as: $H_{0,\text{Variability}}: S_{\text{GDB}} \leqslant \sigma$. This is a new basic hypothesis of the method and also a crucial one because, as stated before, quality control is essentially concerned with variation. The numerical result proposed by Eqs. (5a) and (5b) is (n-1) times the ratio between the observed variability S_{GDB} and the supposed σ. If this expression is significatively greater than n-1 it implies that there is a greater variability in sample and, in consequence, $H_{0,\text{Variability}}$ is rejected and causes of this situation should be analyzed and eliminated in order to bring the process under control. On the contrary $H_{0,\text{Variability}}$ is accepted and it implies that $S_{\text{GDB}} = \sigma$, or that there is a lower variability in sample $(S_{\text{GDB}} < \sigma)$ which implies better quality in the GDB than expected. Here the process is under control but causes can also be analyzed for discovering good practices. The last is very important for a producer because it is, some what, related to a measure of the quality of the process

through the so-called process capability index: a measure of the ability of the process to produce products that meet the specifications (Montgomery 2001).

Development of Acceptance Curves

In this section we show the methodology that, based on simulation, has been developed to obtain an operating characteristic curve for an acceptance process based on the proposed control methodology. The objective is to obtain a tool that will allow us to decide on the acceptance or not of a subcontracted GDB from a positional point of view. Against the simple acceptance or rejection derived from any positional test, this method informs us about the assumed levels of risk (user and producer) and about the size of the sample that has to be adopted to reduce those levels of risk. As previously commented, the process has been developed solely for planimetry. The simulation methodology comprises two main steps:

1. Simulation of populations. Synthetic populations of well known parameters are derived from a controlled statistical random values generation process. Population values are considered to be positional error values.
2. Simulation of samples. By means of a bootstrapping process, samples of different sizes are extracted from each population. Our method is applied to each sample as if it were a single positional control test, but results are aggregated.

The process begins with the generation of random populations of normal distributed values [N ($\mu_P=0, \sigma_P^2=1$), where "P" means population] according to the Box–Muller method (Box and Muller 1958). The fulfillment of the hypothesis is verified by means of an adherence test to the Normal distribution (Kolmogorov–Smirnov) and a test of randomness (Wald–Wolfowitz) (Caridad 1985).

By the previous statistical method two populations P_x and P_y are created, each one with $N_{Px}=N_{Py}=1{,}000$ values. Both of these, P_x and P_y, work as a database of simulated planimetric errors for the components X and Y. From the previous populations, and for each planimetric component, i different p samples are obtained at random, each one with m elements ($m=10,20,30,40,\ldots$), denoted $p_x(i)_m$ and $p_y(i)_m$, respectively. Both of them, $p_x(i)_m$ and $p_y(i)_m$ are treated as if they were the errors found in i different samples of 10, 20, 30, … up to 500 well-defined points measured in a GDB and in a more precise survey. For the robustness of the process i takes a large value, in this case $i=1{,}000$.

The proposed test will be applied to all of these i samples to determine the acceptance or rejection of the product from a positional point of view. Previous to using the test it is necessary to fix its threshold values: (1) global significance level ($\alpha=5\%$); (2) absence of systematic errors ($\mu_x=\mu_y=0$); and (3) assumed dispersion behavior ($\sigma=1, 0.95, 0.9, \ldots$) of the population. Fig. 1 shows the flow diagram of the sample's simulation process. Because of the previous process's depending on only one synthetic population, in order to ensure the statistical robustness of the whole process a new simulation step is considered. Now we generate j random populations $P_x(j)$ and $P_y(j)$, each one with 1,000 normal typified distributed values using the techniques and controls previously described. Fig. 2 shows the flow diagram of the population's simulation process where the sample's simulation process is included.

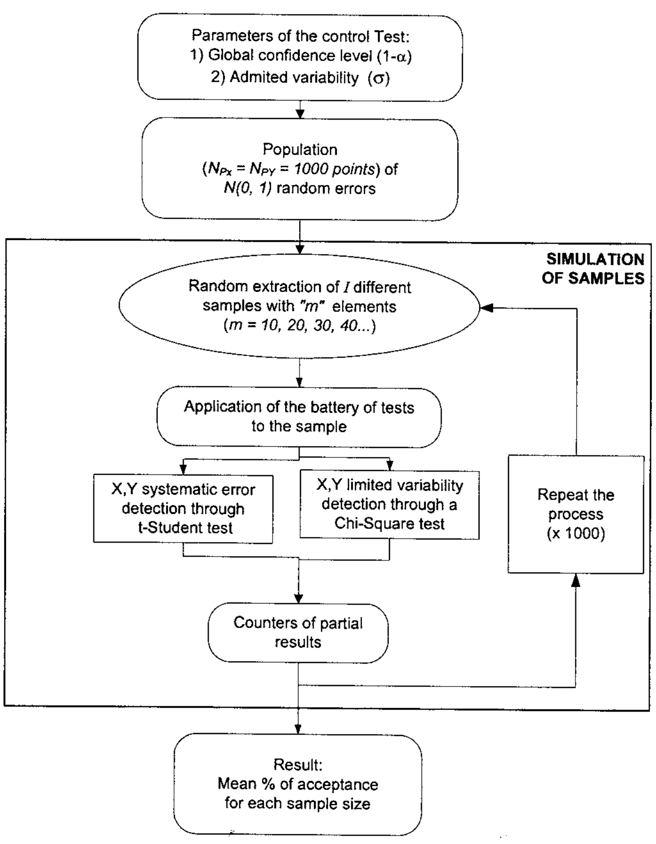

Fig. 1. Flow diagram of the sample simulation process (partial process simulation)

Results

The main result of the simulation is a family of curves that can be expressed graphically as is shown in Fig. 3, but also analytically.

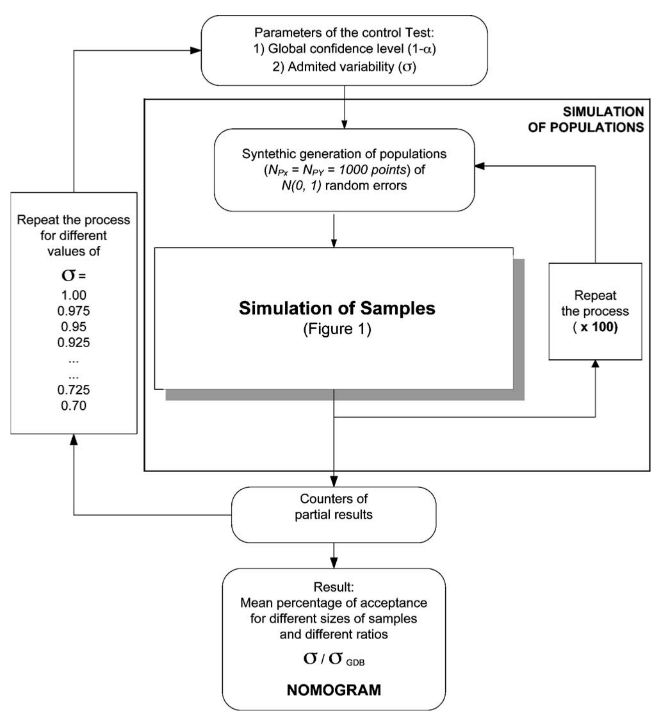

Fig. 2. Flow diagram of the population simulation (global process of simulation)

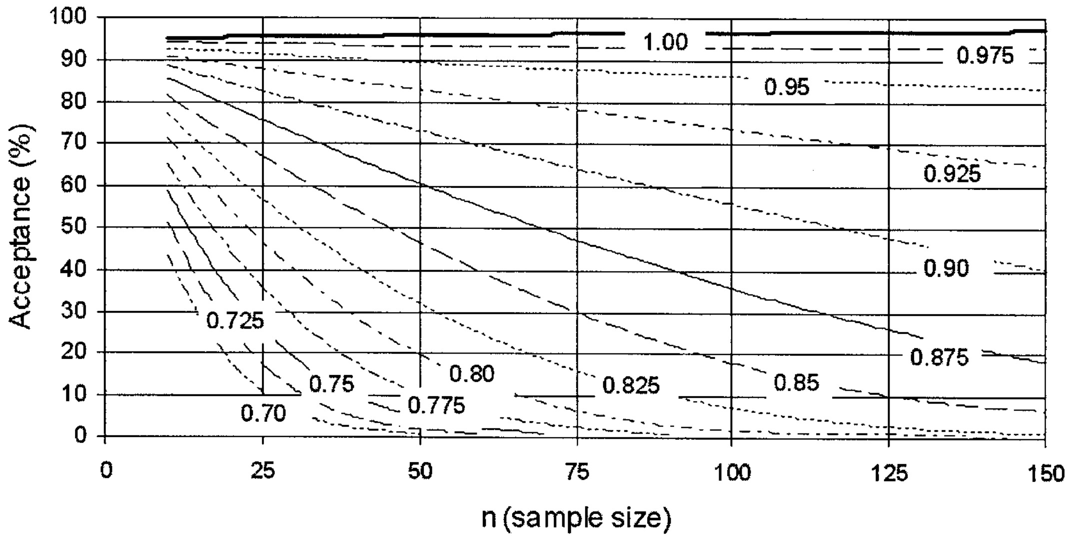

Fig. 3. Nomogram; acceptance curves for ratios $\sigma/S_{GDB}=0.7;0.75;\ldots,0.95;1$ for different size of samples $n\in[10,150]$

We call the graphic expression a nomogram. These curves are only controlled by the variational features of the GDB in relation to the expected or supposed one, and bias does not affect these curves because it can be removed. These curves show the power of the battery of statistical tests applied together. For a given ratio between the actual deviation of the GDB (S_{GDB}) and the deviation assumed in the control (σ), the nomogram represents the evolution of acceptance levels in percentage, depending on the number of control points used. In a certain way the ratio σ/S_{GDB} is similar to the capability of a process in the statistical control of industrial processes.

The ratio σ/S_{GDB} takes values greater or equal to one when the variation of data of the GDB is lesser than the variation considered in the control ($\sigma\geqslant S_{GDB}$), which means, in other words, that the product has enough quality, or that the process is under control. For this case the nomogram shows a curve which is practically horizontal (line labeled with 1.0) at 95% acceptance level ($Al=95\%$). The interpretation is easy, the product is good enough and it is accepted in 95% of control cases. There is no risk for the user but a 5% producer's risk. The product is good but it is rejected. This is the Type I error which occurs in 5% of the cases (the significance level). The 95% acceptance level is derived from the global significance of the complete test. Of course if $\sigma \geqslant S_{GDB}$ the producer's risk will decrease but producer and user must consider the potentially higher cost of such process.

The ratio σ/S_{GDB} takes values less than one when $\sigma\leqslant S_{GDB}$, which means that the product does not have enough quality, or that the process is out of control. For this case the nomogram shows several curves, labeled with the deviation ratio (0.975; 0.95; 0.925; …). These curves show the evolution of the product's acceptance levels depending on the size of the sample. Now the interpretation is that the product is not good enough, but it is accepted in a variable percentage of cases depending on the size of the control sample. Thus, the acceptance level acts here as a user's risk because a bad product is accepted on average as many times as the acceptance level points out.

Regarding the nomogram's curves, it can be observed that the lower the size of the sample the closer these curves are placed and also the higher values of acceptances are obtained. This indicates higher uncertainty but also a situation that favors the producer's position. Obviously, if we desire a higher level of security, and also to reduce the user's risk to a desired level, we should increase the size of the control sample.

Another interesting characteristic of the nomogram is that the stability of its curves is very high because of the process of simulation itself, in which a high number of simulations have taken place. The represented curves give a maximum variability of ±0.75%. The nomogram has been derived by a simulation process in which deviation of population has always been set to one ($S_{GDB}=\sigma_P=1$), and the deviations σ considered for the control (Steps 6 and 8) have taken different values from 0.7 to 1, with a step of 0.025 units. For this reason, the nomogram can be considered typified and valid for any value. So generic acceptance curves have been achieved for different σ/S_{GDB} ratios and sampling sizes.

Let us observe two examples of the use of the nomogram, when there is or is not a situation of user risk:

Case 1: Consider we have a GDB for a vegetation change analysis study and that what we consider an acceptable uncertainty for this use (product specifications) is a variability of 4 m ($\sigma=4$ m). For the positional control of the planimetry we develop a GPS field survey from which we obtain a sample of $n=25$ well-defined points. This sample characterizes statistically the product with the following values: $\mu_{GDB}\approx 0$ m (there is no bias or it has been adequately eliminated); $S_{GDB}=3.5$ m. Now we have to typify our values in order to enter into the nomogram. This is achieved by computing the ratio $\sigma/S_{GDB}=4/3.5=1.143$. This value is greater than 1 and means we have data of good quality because the maximum acceptable error is 14.3% greater than the error we have assessed. In this case we are over the horizontal line of the nomogram which is labeled with A. This means we are protected by the quality of our data independently of the sample size. There is no user risk, the producer has a process with enough capacity and here the only question is the cost of the over-quality production for the producer and user.

Case 2: Consider we have the same GDB as the previous example but now the proposed use is a little more exigent and tolerates an uncertainty no more than 3 m ($\sigma=3$ m). Also we use the same GPS field survey as in the previous case. In order to typify our values we compute the ratio $\sigma/S_{GDB}=3/3.5\approx 0.85$. This value is lesser than 1 and means you have data of poor quality because the maximum acceptable error is 15% (1.0

$-0.85 = 0.15$) less than the error we have actually assessed. In this case we have to look in the nomogram for a curve labeled with this value (0.85), or interpolate it. This means we are in a risk (user risk) whose probability depends on the sample size used to assess the positional accuracy of our data. For our case where $n = 25$ this curve shows approximately 68% of acceptances which actually is a high risk for the user, because the quality of the GDB is poorer than expected. If we wish to reduce this risk to a lower value, let us say 5%, the nomogram indicates that we need to use at least 150 control points. As is shown in both examples, the use of our methodology always implies the use of our own estimate of what is an acceptable error or of the specifications of the product being controlled.

The previous examples are based on a single sample, and allow us to obtain one-point estimation on the nomogram. A more appropriate use of the nomogram can be realized if bootstrapping techniques are applied in order to obtain a robust estimation of the statistical parameters of the sample and, in this way, of the acceptances. Here the process is similar to that described in the second step (simulation of samples) of the proposed methodology for the development of the acceptance curves. The control sample of size n works as population, and the bootstrap allows us to obtain for each different sample size n', with $n' < n$, an estimation of the acceptance, which actually is a function of n'. These values of acceptances can be plotted on the nomogram, defining a curve which will show the evolution of the expected acceptance for that GDB under the production process that the producer employs for a given product.

From our point of view, this methodology and its expression in a family of curves have some important advantages for positional control:

- They allow user and producer to learn the capacity of the process;
- They make explicit to the user the type II error level for a wide range of σ/S_{GDB} ratios and sample sizes;
- They indicate in a very clear fashion the high level of user risk when working with small sample sizes;
- They inform the user of the sample size needed in order to reduce risk to a desired level.

Conclusions

The concept of operating characteristic curves, which is used in industrial statistical quality control, has been applied to a specific positional-planimetric control test, but it could also be applied successfully to other positional control methodologies. All the development set out in previous sections is based on statistical theory, that is to say that results are valid as long as basic hypotheses are followed.

Through a simulation process and a bootstrap procedure, we have derived a family of curves, or nomogram, which shows the evolution of acceptance levels depending on the number of control points used and on the ratio between the deviation σ assumed for the control and the actual deviation of the GDB, S_{GDB}. The ratio σ/S_{GDB} exerts a strong control over the acceptance process and plays a role similar to a process capability ratio in an industrial process.

The nomogram has been obtained by means of simulation with typified values, so that it can be considered typified, or general, because it depends only on the ratio of deviations between the deviation established for the control and that actually observed from the GDB. The result tells us the risk level that the user assumes by accepting a GDB but also informs us about the size of the control sample that should be used to reduce this risk level to a value that the user considers satisfactory. As a final remark, it is our opinion that the application of concepts of industrial statistical control to spatial data can give a great deal of insight to a very important cartographic process.

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Notation

The following symbols are used in this paper:

e = error;
n = simple size;
S = observed standard deviation from a sample;
T = value of the t-student distribution;
x = east coordinate or component;
y = north coordinate or component;
α = significance level (commonly 5%);
β = Type II error;
μ = mean observed error value from a sample;
σ = supposed standard deviation for a product or population;
χ^2 = value of the Chi-square distribution.

Subscripts

i = the ith case in the sample;
OBS = observed (derived from a sample of the product);
True = field survey;
X = east coordinate or component; and
Y = north coordinate or component.

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